Criteria for the Design of Loop-Type Directional Couplers for the L Band*

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Summary—**For many years the design of loop type directional couplers has relied heavily on experiment. One of the most common varieties has been the loop coupler much shorter than a wavelength and having a built-in termination (vestigial arm). In this paper the criteria for the design of loop couplers of any length are considered, and their application to either coaxial line or waveguide are discussed. The theoretical basis for design is established by first considering the theory of coupled transmission lines. Several designs which utilize a quarter-wavelength or shorter loops and which have very desirable features with regard to coupling, directivity, and bandwidth are illustrated. Typical performance of couplers having couplings of 20–40 db and directivities of over 30 db are presented.**

Introduction

By a loop type directional coupler is meant a directional coupler consisting of a segment of an auxiliary conductor located within either a coaxial line or a waveguide and brought out through the wall to the desired connectors or terminations. A loop type coupler may be thought of under some circumstances as a prolongation or segment of a secondary transmission line. The plane of the loop is normally approximately parallel to the axis of the main line to which the loop is coupled. All loop type couplers when properly constructed have intrinsic backward directivity; that is, the coupled energy in the secondary line travels in the opposite direction from that taken by the wave in the main transmission line or guide.

Although loop type directional couplers have found wide application for years in the measurement of standing wave ratio, reflection coefficient, and output power on commercial vhf, uhf, and microwave transmitters, their design has been largely an effort of cut-and-try. The literature, which is quite extensive on directional couplers in general, has been rather sparing in telling how one should go about designing loop type directional couplers. In the recent literature, Allan and Curling,[1] Monteath,[2] Oliver,[3] Firestone,[4] Knechtli,[5] and Wolf[6,7] have pointed out a number of features in the analysis and performance of loop type directional couplers. It is the purpose of this paper to bring together a number of these points and present the criteria which must be considered in actual design. It should be pointed out that cut-and-try is not eliminated, but has been given direction.

Several approaches may be used to analyze the loop type coupler. When the loop is of substantial length compared to a wavelength, we may apply the theory of multiconductor transmission lines. As the length of the coupler is reduced we reach a point (around $\lambda/12$) where lumped circuit analysis may be conveniently applied, the parameters of the equivalent circuit being simply related to the distributed parameters of the previous case. The advantage of this lies primarily in the ease of manipulation leading to results which *check* the much more formidable transmission line approach. Finally as the length of the loop is made still shorter, it may be necessary to abandon both approaches due to the difficulty in specifying the parameters.

The Transmission Line Approach

Starting from the well-known concepts of Maxwell's coefficients of capacity, induction, and potential in the case of a system of charged conductors, and their counterparts, the coefficients of self and mutual inductance in the case of a set of current carrying wires, the generalized telegraphers equations for a multiconductor transmission line can be derived. This has been done in a classical paper by Carson and Hoyt,[8] who were primarily concerned with cross talk on telephone lines. Later, Bewley[9] and Pipes[10] again treated the general equations, but from the viewpoint of studying surges on power transmission lines. Recent writers such as Oliver,[3] Knechtli,[5] and Wolf[6,7] treated the equations from the viewpoint of directional coupling effects.

Starting from the concepts cited we are led to the set of differential equations (1) representing the voltage and current situation on two coupled transmission lines

* Manuscript received by the PGMTT, July 16, 1956. Presented before the National Symposium on Microwave Techniques, Philadelphia, Pa., February 3, 1956. This work was done at the Moore School of Elect. Engrg. under AF Contract No. AF 30(635)-2801 with the Rome Air Dev. Ctr., Rome, N. Y.

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with a common return conductor (see Fig. 1). Here V_1 and V_2 are the respective potential differences on the two lines; I_1 and I_2 are the respective currents; C_1, C_2, L_1, L_2 are the self parameters which it may be shown are respectively Maxwell's capacitance coefficients and the coefficients of self inductance; C_{12} and L_{12} are the corresponding mutual parameters.

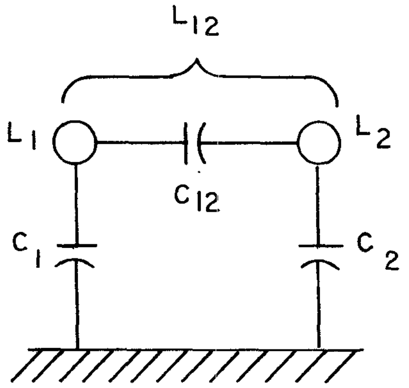

Fig. 1—Geometry of lines described by (1).

$$\frac{\partial V_1}{\partial x} = -j\omega L_1 I_1 - j\omega L_{12} I_2$$
$$\frac{\partial V_2}{\partial x} = -j\omega L_2 I_2 - j\omega L_{12} I_1$$
$$\frac{\partial I_1}{\partial x} = -j\omega C_1 V_1 - j\omega C_{12} V_2$$
$$\frac{\partial I_2}{\partial x} = -j\omega C_2 V_2 - j\omega C_{12} V_1. \quad (1)$$

The solution of these equations yields not only a pair of forward and backward traveling waves as in ordinary transmission lines, but rather two such pairs (or normal modes) having, at least theoretically, different velocities as shown by (2).

$$V_1 = A_{11}\epsilon^{-\gamma_1 x} + A_{12}\epsilon^{-j\gamma_2 x} + A_{13}\epsilon^{+j\gamma_1 x} + A_{14}\epsilon^{+j\gamma_2 x}$$
$$I_1 = B_{11}\epsilon^{-j\gamma_1 x} + B_{12}\epsilon^{-j\gamma_2 x} + B_{13}\epsilon^{+j\gamma_1 x} + B_{14}\epsilon^{+j\gamma_2 x}$$
$$V_2 = A_{21}\epsilon^{-j\gamma_1 x} + A_{22}\epsilon^{-j\gamma_2 x} + A_{23}\epsilon^{+j\gamma_1 x} + A_{24}\epsilon^{+j\gamma_2 x}$$
$$I_2 = B_{21}\epsilon^{-j\gamma_1 x} + B_{22}\epsilon^{-j\gamma_2 x} + B_{23}\epsilon^{+j\gamma_1 x} + B_{24}\epsilon^{+j\gamma_2 x}. \quad (2)$$

The values of the new propagation constants γ_1 and γ_2 are defined by (3).

$$\gamma^4 - 2\gamma^2\beta_0^2\left[k_L k_C + 1 + \frac{(h^2-1)^2}{2h^2}\right] + \beta_0^4(1-k_C^2)(1-k_L^2) = 0 \quad (3)$$

where

$$k_L = \frac{L_{12}}{\sqrt{L_1 L_2}}$$

$$k_c = \frac{C_{12}}{\sqrt{C_1 C_2}}$$

$$\beta_0 = \sqrt{\beta_1\beta_2} = \omega(L_1 C_1 L_2 L_2)^{1/4}$$

$$h = \sqrt{\frac{\beta_2}{\beta_1}}.$$

The constants k_C and k_L are measures of the capacitive and inductive couplings, and a new mean phase constant β_0 and a phase constant ratio h are defined. In general $\gamma_1 \neq \gamma_2 \neq \beta_1 \neq \beta_2$. This concept of multivelocity waves has been mentioned in the literature (*e.g.*, Bewley[9]), but under many conditions, *e.g.*, widely spaced open-wire lines, has been ignored due to the very slight differences between the constants in question.

For most cases β_1 and β_2 do not greatly differ, and under these conditions $\gamma_{1,2}$ is given by (4). This obviously *does not* include cases such as the coupling of a coaxial line with a hollow waveguide.

$$\gamma_{1,2} = \beta_0\sqrt{(1 \pm k_C)(1 \pm k_L)}. \quad (4)$$

The sixteen integration constants shown previously in the solutions for V and I can be reduced to four, and these four are determined by the boundary conditions. Assuming the condition of (4), a generator voltage of 2 units in line 1, and termination of each line by its characteristic impedance we obtain the results shown in (5a) and (5b).

$$V_F \cong -j\sqrt{\frac{Z_{02}}{Z_{01}}}\sin\left[\beta_0\left(\frac{k_C + k_L}{2}\right)l\right]\epsilon^{-j\beta_0 l} \quad (5a)$$

$$V_B \cong +\sqrt{\frac{Z_{02}}{Z_{01}}}\left(\frac{k_L - k_c}{4}\right)\left[1 - \cos\{\beta_0(k_C + k_L)l\}\epsilon^{-j2\beta_0 l}\right]. \quad (5b)$$

The voltages coupled into the secondary line are dependent on the ratio of characteristic impedances, the capacitive and inductive coupling, and the length. Two TEM transmission lines in close proximity have k_C and k_L naturally of the same order of magnitude and of opposite sign. We call the resulting coupling "intrinsic backward coupling." Perfect (∞) directivity occurs when $k_C = -k_L$. This leads to the so-called "characteristic equation" $L_{12}/C_{12} = Z_{01}Z_{02}$. Usually, however, this condition is not obtained exactly, and we have the expressions for directivity and coupling as shown in (6) and (7).

$$C = -20\log_{10}\left(\frac{k_L - k_C}{2}\right)\sin\beta_0 l \quad (6)$$

$$D = 20\log_{10}\frac{k_L - k_C}{k_L + k_C}\,\frac{\sin\beta_0 l}{\beta_0 l}. \quad (7)$$

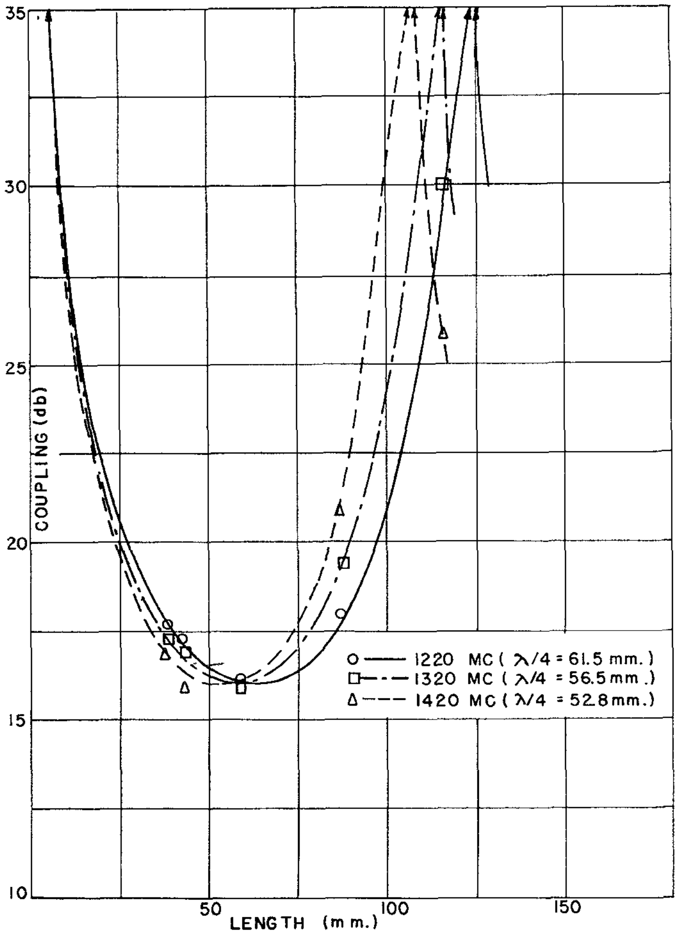

Fig. 2—Variation of coupling with length for a loop coupler (copper strip 9 mm in width penetrating 5 mm into RG-46/U coaxial line).

The coupling varies sinusoidally with length and thus is a maximum when the length of the coupling region or loop is a quarter-wavelength. Experimental verification of this for several frequencies is shown in Fig. 2. It is noted that in accordance with the conventional definition of coupling *a small number of db* means *a larger amount of coupling.*

When the length of the loop is $\frac{1}{4}$ wavelength the coupling is virtually insensitive to frequency. However as the length is reduced toward zero a greater variation is obtained, reaching a maximum slope of 6 db per octave when the loop is very short. This variation of coupling within a typical design band, with physical length is shown in Fig. 3.

The dependence of the directivity on $|k_L/k_C|$ or $|k_C/k_L|$ is shown in Fig. 4. It can be seen that the directivity goes up very rapidly as the condition for perfect equality of $|k_C|$ and $|k_L|$ is approached.

Whether $|k_C|$ or $|k_L|$ is the predominant factor can be determined if the loop is rotatable according to the method described by Allan and Curling.[1]

The directivity and coupling expressions both contain $\sin \beta_0 l$, and $(k_C - k_L)$. Consequently there is an interdependence between them as shown in Fig. 5 where

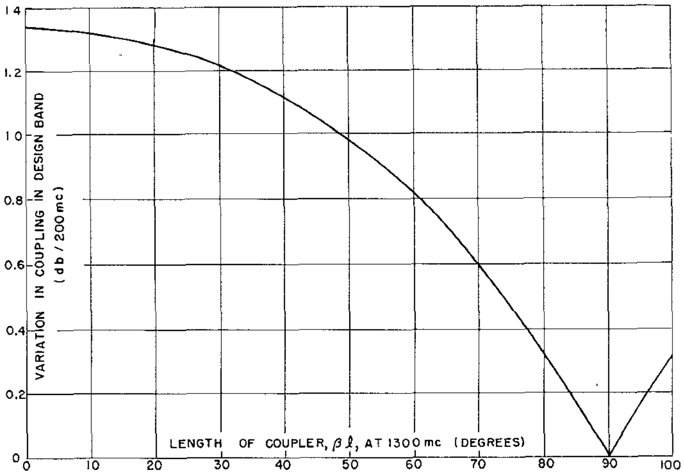

Fig. 3—Variation of coupling within 200 megacycle design band for different coupler lengths.

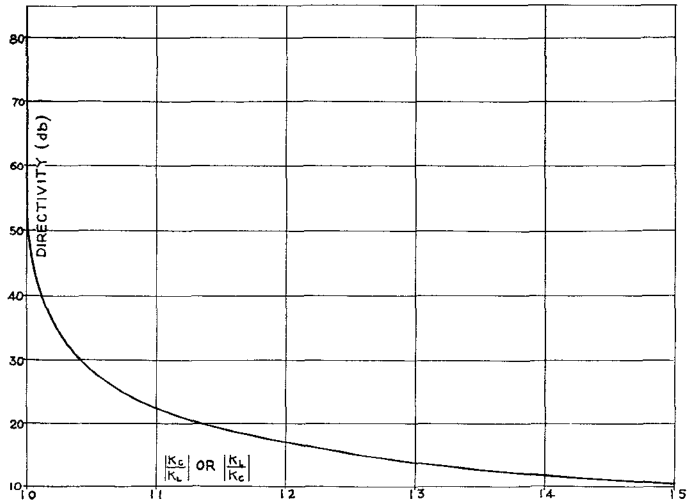

Fig. 4—Dependence of directivity upon $|k_c/k_L|$ or $|k_L/k_c|$ for quarter-wave loop coupler.

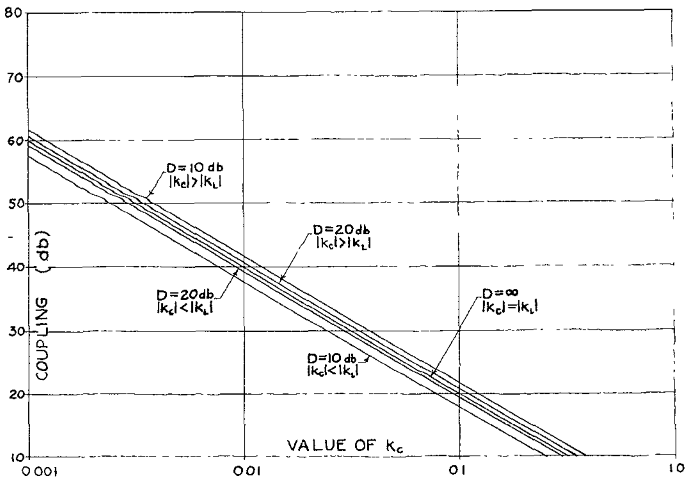

Fig. 5—Dependence of coupling upon k_c for various directivities for quarter-wave loop coupler.

coupling is plotted vs k_c for various conditions of directivity. This graph shows that the same directivity may be obtained for different combinations of k_L and k_C; however these *same* values of directivity yield *different* values of coupling. This curve is useful for estimating the tolerance which must be placed on directivity to ensure coupling within certain limits. It should be noted, however, that it assumes "perfect" termination of the secondary line (vswr <1.02).

Coupler Design for Coaxial Line

In designing directional couplers according to the general concepts outlined above, the principal stumbling block is that k_C and k_L cannot be theoretically predicted with great accuracy for most configurations. Furthermore the condition for $|k_C| = |k_L|$ is met by a number of different combinations of auxiliary conductor size and spacing, each one of which has a different characteristic impedance, and each one of which demands a different termination. Firestone[4] has shown that the termination on line 2 need not necessarily be equal to Z_{02}, but must then have a conjugate relationship to the termination on line 1 if high directivity is to be preserved. In the microwave region good terminations for values other than 50 ohms are not too common; usually also, the main line Z_0 is 50 ohms. This restricts the problem severely, and will be the matter first discussed.

In the necessary empirical part of the problem the first step is to secure a matched auxiliary line (vswr <1.02) when looking into the output terminal, the other end being properly terminated in a "good" termination. When the conductor spacing has been adjusted to give this condition to a given conductor, the coupling and directivity are checked. Then a new combination of conductor size and spacing is tried. In a relatively short time the high directivity needed can be secured. The coupling can then be secured by calculation of the needed length from the formula given. Of course, one is restricted as to the magnitude of coupling one can secure by this means. Fig. 6 shows the configuration and the results of a procedure such as this.

This procedure has been successfully used to design a directional coupler with four outputs as shown in Fig. 7. This device was made for use in an L band system utilizing RG-46/U, $1\frac{5}{8}$ inch, transmission line. The lengths of the loops were chosen to give a series of coupling values between 20 and 30 db. The directivity for each was over 30 db in the range 1200–1400 mc.

For smaller values of coupling (*i.e.*, actually tighter coupling) one must use a higher impedance secondary line requiring usually a vestigial arm (or built-in) termination. The empirical procedure would be much the same, only the condition of match in the secondary line cannot usually be conveniently determined by slotted line methods. It is useful to provide a tuning arrangement on the vestigial arm termination which has the effect of allowing adjustment of the reactance of the load and the loop itself, so that high directivity is obtained. See Fig. 8 for an example.

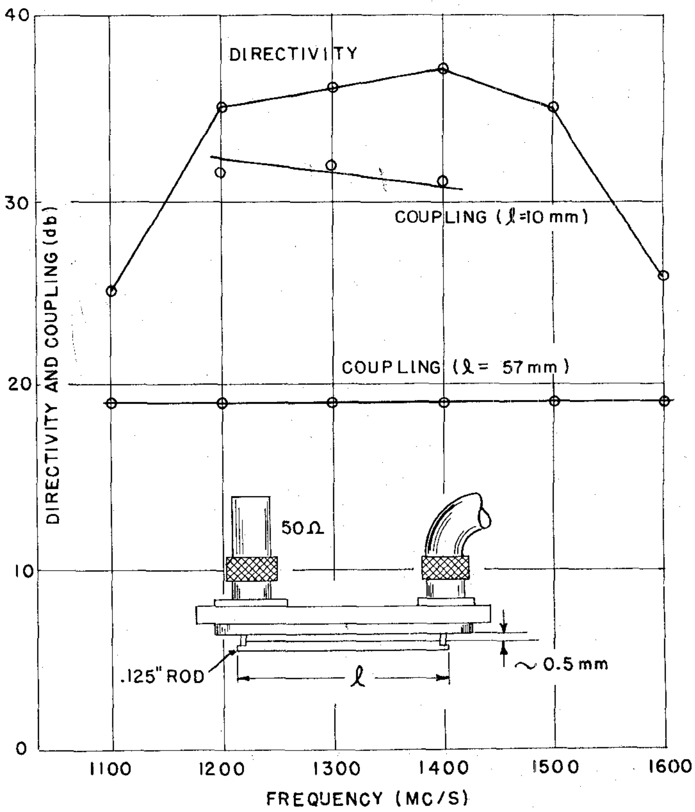

Fig. 6—Directivity and coupling of 50Ω loop type couplers of two different lengths applied to RG-46/U coaxial line.

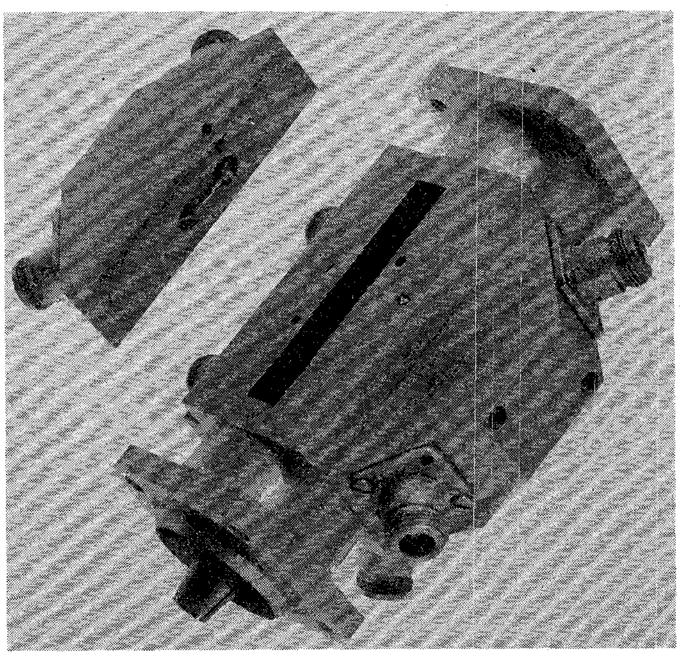

Fig. 7—Four-output directional coupler in RG-46/U coaxial line.

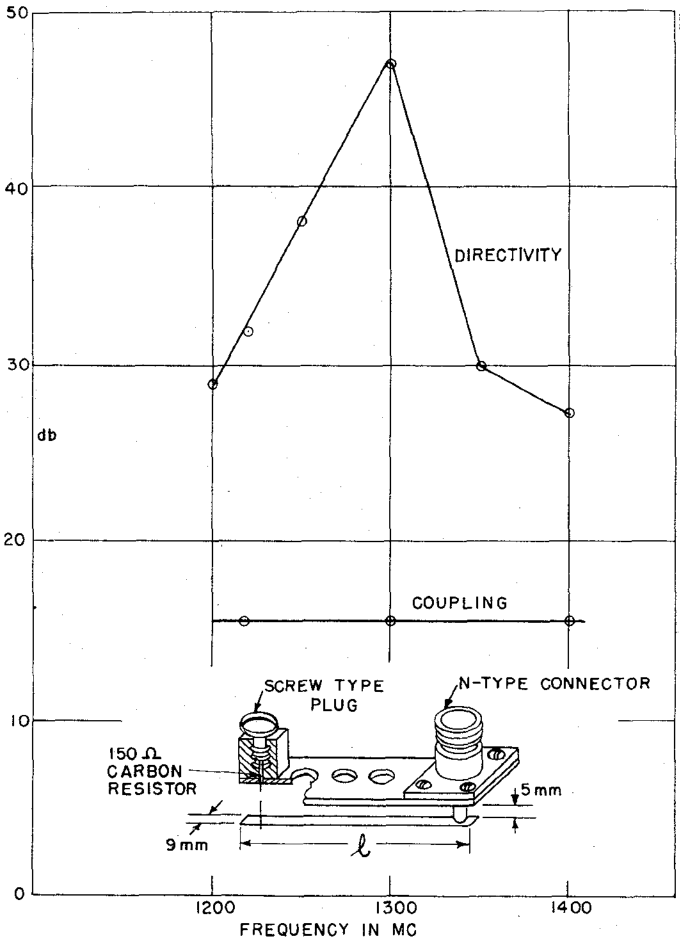

Fig. 8—Performance of quarter-wave loop coupler with vestigial arm termination for RG-46/U coaxial line.

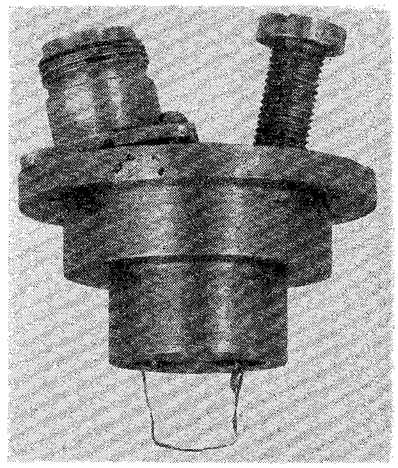

Fig. 9—Vestigial arm loop coupler for RG-69/U waveguide.

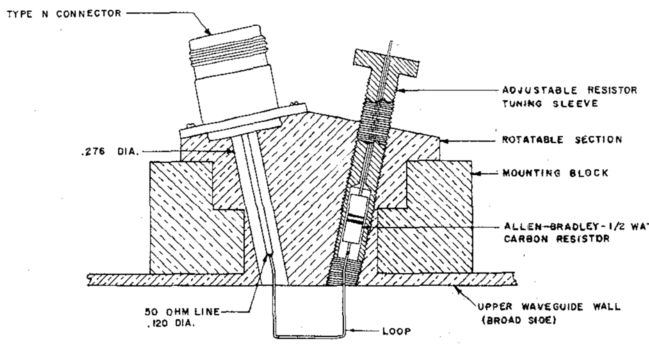

Fig. 10—Constructional details of vestigial arm loop coupler for RG-69/U waveguide.

Waveguide Loop Couplers

The theory of coupled transmission lines laid down above can be applied to the problem of coupling a coaxial line with a waveguide up to a point. However, the fact that the phase velocities in the two media are so different, and the difficulties in suitably defining the parameters of a waveguide, make the further application of the theory impractical.

Exhaustive experiments with coupling a coaxial line "naturally" through the broad wall of rectangular waveguide have shown that "intrinsic backward coupling" occurs for only one condition (extremely loose coupling), and is very frequency dependent due to the dependence of phase velocity upon frequency in the waveguide. This implies that loop couplers in waveguides based upon coupled transmission effects are not very practical.

However, if a very short loop is used, it no longer acts as a transmission line. It may be shown, however, that the perfect directivity condition given previously, *i.e.*, $C_{12}/L_{12}=Z_{01}Z_{02}$ is still valid, although some points of argument can be raised about the proper definition of the quantities involved.

Practically, a vestigial arm type of loop with a tuning device, as previously described, is used. Figs. 9 and 10 show one such coupler which was used in RG-69/U, L band waveguide. Each desired value of coupling requires a different area and consequently a different terminating resistance. For each condition selected, the "tuning sleeve" must be adjusted for optimum directivity. By this process fair results for coupling, directivity, and bandwidth were obtained for the L band as shown in Fig. 11.

It was found that a relationship exists between area of the loop and the coupling, nearly independent of the size and type of conductor used. This relation is presented in Fig. 12. The theoretical plot in the figure is obtained through the following approach.

The mutual inductance of a loop in the center of a rectangular waveguide excited in the dominant mode may be written

$$L_{12} = \mu_0 \frac{2A}{al} \tag{8}$$

where A is the area of the loop, a, the broad side of the guide, and l the length of the coupler. In the case of a short loop the coupling parameter will be defined as

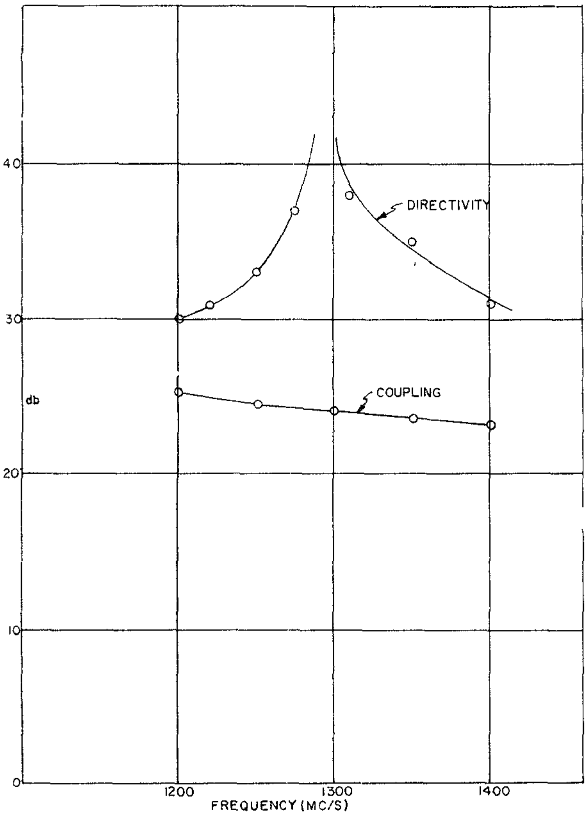

Fig. 11—Performance of vestigial arm loop coupler for RG-69/U waveguide.

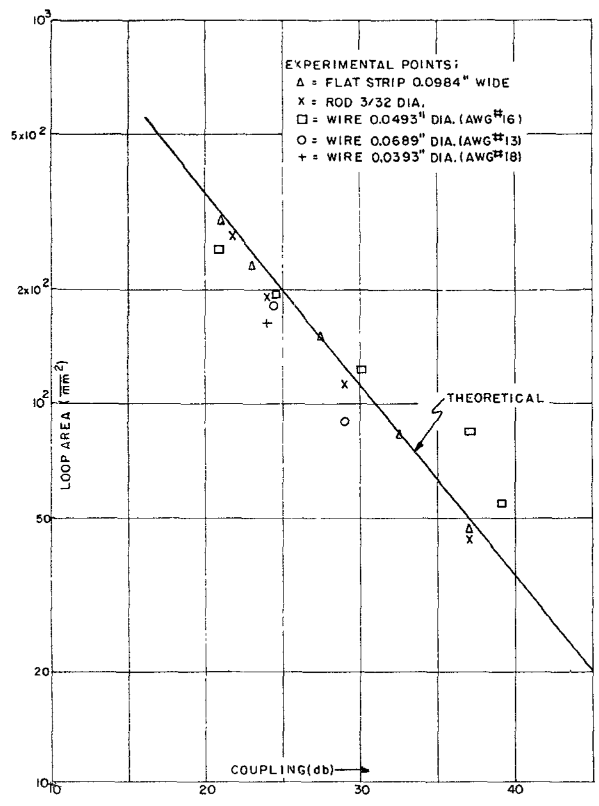

Fig. 12—Loop area—coupling relationship for vestigial arm loop coupler for RG-69/U waveguide. f=1300 mc.

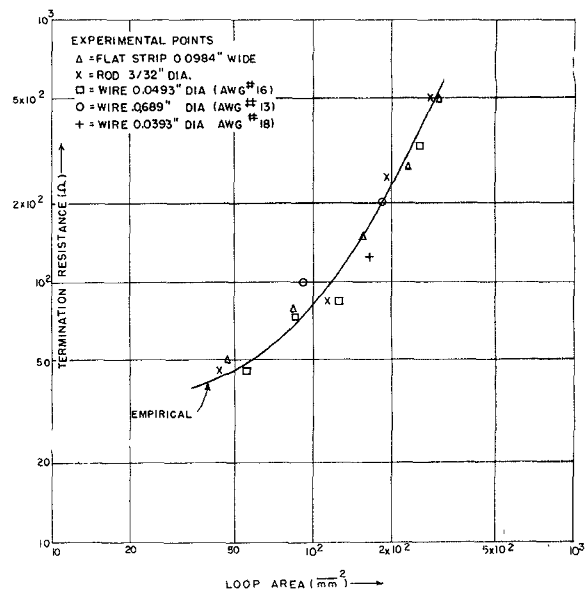

Fig. 13—Necessary loop area—terminating resistance relationship for vestigial arm loop coupler for RG-69/U waveguide. f=1300 mc.

$k_L = L_{12}/L_1$ where L_1 is the equivalent distributed inductance for unit length of the waveguide which is taken to be

$$L_1 = \mu_0 \frac{2b}{a} \cdot \tag{9}$$

This expression for k_L is the same as that mentioned previously, $k_L = L_{12}/\sqrt{L_1 L_2}$ where $L_2 = L_1$, a condition that appears to be reached by adjustment of the "tuner," provided L_2 is initially large enough. No reason other than the fair agreement shown in Fig. 12 can be given here, for L_2 is neither accurately measurable nor calculable.

Using (6) with $|k_C| = |k_L|$ the coupling is

$$C = 20 \log \frac{b\sqrt{\lambda_g \lambda_0}}{2\pi A} \cdot \tag{10}$$

Eq. (10) is plotted in Fig. 12, for an RG-69/U waveguide, and a frequency of 1300 mc.

Each value of area selected demands a unique value of terminating resistance for preservation of good directivity. The empirical relationship is shown in Fig. 13.

When designing an L band loop coupler, use may be made of Figs. 12 and 13. For any required value of coupling, Fig. 12, or (10), gives the area of the loop to be used. Entering the curve of Fig. 13 with the above area, an indication of the order of magnitude of the resistor to apply in the vestigial end of the coupler is obtained. Note that the resistor should be "tuned" in the receptacle; *i.e.* the distances of the resistor with respect to the short-circuit, and of the plunger with respect to the loop (Fig. 10) should be chosen so as to have a maximum of directivity and bandwidth.